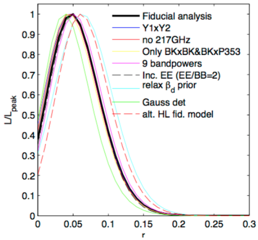
Beyond the CMB: the Effective Field Theory of Large Scale Structure

Ashley Perko
Stanford University

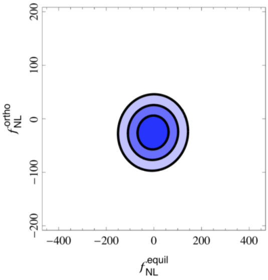
in collaboration with M. Lewandowski and L. Senatore

testing inflation

- to probe inflation: measure r or f_{NL}
- CMB constraints on f_{NL} will not be improved much after Planck
- want to get to $f_{\rm NL} < 1$ to test slow-roll inflation



BICEP2/Keck, Planck Collaborations (2015)



Planck Collaboration (2015)

first, a brief digression into B-modes

what can B modes tell us?

- B modes sensitive to tensor fluctuations during inflation
- "smoking gun" for inflation: can we make this more precise?
- inflation is the only single-field model that can produce scale-invariant scalar modes
- similar no-go theorem for tensors?

Baumann, Senatore, Zaldariagga (2011)

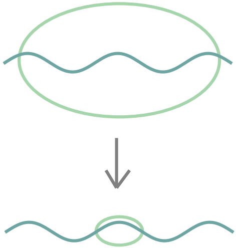
parameterizing the approach to inflation

- parameterize background solutions as a power law: $a = (t/t_0)^\alpha$, so $H = \alpha/t$.

- "slow roll parameter" $-\dot{H}/H^2 = 1/\alpha$

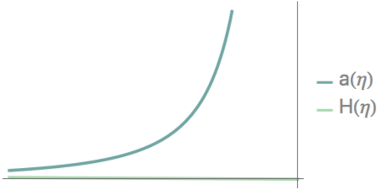
- $\alpha \to \infty$ is de Sitter

- to solve horizon problem, need k/aH to decrease with time

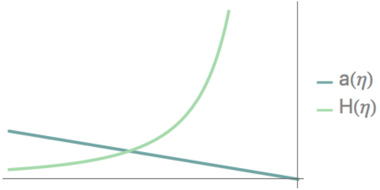


scaling solutions to the horizon problem

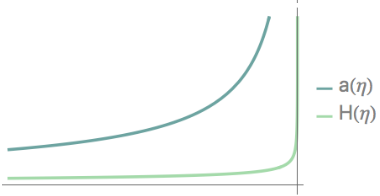
- "not-so-big bang": $\alpha > 1$



- contraction: $0 < \alpha < 1$



- "starting the universe": $\alpha < 0$
  Creminelli, Luty, Nicolis, Senatore



tensors in EFT of inflation

- epoch that pushes modes outside horizon ends to give normal expansion $\Rightarrow$ time diffs spontaneously broken

- EFT of inflation: most generic action consistent with symmetry

- keeping only terms fixed by background gives $\langle\gamma^2\rangle \sim H(t)^2/M_{\rm Pl}^2$

- with speed of sound, $\langle\gamma^2\rangle \sim H^2/c_\gamma M_{\rm Pl}^2 \Rightarrow$ time-dependent speed of sound can restore scale invariance

$$
\begin{aligned}
S \;=\; & \int d^4x\sqrt{-g}\,\frac{1}{2}M_{\rm Pl}^2\left[R^{(4)} - 2(3H^2+\dot{H}) + 2\dot{H}\delta g^{00}\right. \\
& \left. - \left(1 - \frac{1}{c_\gamma(t)^2}\right)\left(\delta K^\mu{}_\nu \delta K^\nu{}_\mu - \delta K^2\right)\right]
\end{aligned}
$$

scale-invariant tensors

- with speed of sound $c_\gamma \sim t^m$, canonically normalized action for helicity modes σ with $dy = (c_\gamma/a)dt$ is

$$S_\sigma = M_{\rm Pl}^2 \int d^3x dy \; y^{2n} \left(\gamma_\sigma'^2 - (\partial_i \gamma_\sigma)^2\right) \; ,$$

$$\text{where } n = \frac{\alpha - m/2}{1 - \alpha - m}$$

- solutions are Hankel functions, scale-invariant if $n = -1 \Rightarrow m = -2$

- fixes time dependence of c_γ

rapidly varying speed of sound

- result independent of α: all scalings of $a = (t/t_0)^\alpha$ allowed?

- because of nonlinear realization of symmetry, couplings that appear in quadratic action also in cubic action with fixed coefficients

- c_γ is very rapidly changing: if e^N modes go outside horizon, c_γ varies by:

$$\frac{c_{\gamma,f}}{c_{\gamma,in}} \sim \left(\frac{t_f}{t_{in}}\right)^{-2} \sim \left(\frac{a_f H_f}{a_{in} H_{in}}\right)^{2/(\alpha-1)} \sim e^{\frac{2N}{\alpha-1}}$$

- but $c_\gamma < 1$ for subliminality, so $c_\gamma \ll 1$ at some point

constraints from weak coupling

- cubic action is large when $c_\gamma \to 0$

$$\frac{L_{\gamma\gamma\zeta}}{L_{\zeta\zeta}} \sim \frac{M_{\rm Pl}^2 a^3 \zeta \dot{\gamma}_{ij} \dot{\gamma}^{ij} c_\gamma^{-2}}{M_{\rm Pl}^2 a^3 \dot{\zeta}^2} \sim \frac{1}{c_\gamma^{5/2}\sqrt{\alpha}} \langle \gamma^2 \rangle^{1/2}$$

- constrained by weak coupling

$$\frac{L_{\gamma\gamma\zeta}}{L_{\zeta\zeta}} \ll 1 \quad \Rightarrow \quad c_\gamma \gg 10^{-2} \left(\frac{r}{\alpha}\right)^{1/5}$$

constraints on tensor modes

- get bound in terms of r and α

$$\frac{N}{|1-\alpha|} \ll 2 - \frac{1}{10}\log\frac{r}{\alpha}$$

- satisfied when $\alpha \to \infty$
- to have a large $\alpha \ll 1$, need non-perturbatively small r
- example: for $r > 10^{-6}$ and $N = 10$, need $\alpha \gtrsim 4$ $(\epsilon_{\rm sl} \lesssim 0.25)$
- these are inflation-like backgrounds

extensions

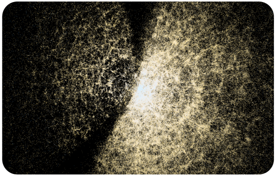
- NEC-volating backgrounds
- particle production mechanisms

AP, Senatore (to appear)

on to LSS

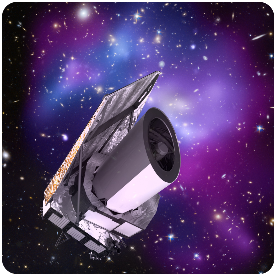
constraining cosmological parameters with LSS

- projection for EUCLID: $\Delta f_{NL} = 3.0$ with $k_{max} = 0.15$
  Giannantonio et. al. (2011)



- these estimates use only linear theory

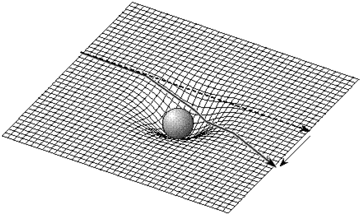
- if we can extend UV reach, number of modes goes like k^3_{max}



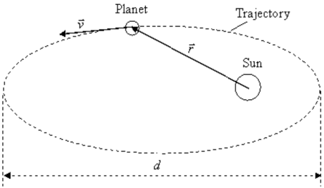
- for $k_{max} \sim 0.3$, this means $\Delta f_{NL} < 1$!

advantages of EFT

- approximation of high energy (UV) theory at low energies (IR) + perturbative corrections
- UV theory is known: integrate out to get simpler IR theory
- UV theory unknown: parameterize ignorance of UV effects in EFT parameters



$\Downarrow$

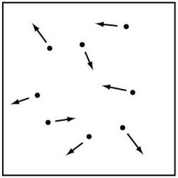


an EFT of LSS

- UV = Boltzmann equation for dark matter particles + Newtonian potential
- IR = effective gravitational fluid
- UV theory is known, so parameters can be calculated and extracted from small-scale simulations
- or, write down generic stress tensor and match to observations

UV Construction for DM

- phase space density $f(\vec{x},\vec{p})d^3xd^3p$: probability that there is a particle in volume d^3xd^3p



- for particles, given by

$$f_n(\vec{x},\vec{p}) = \sum_n \delta^3(\vec{x}-\vec{x}_n)\delta^3(\vec{p}-ma\vec{v}_n)$$

- each particle obeys collisionless Bolzmann equation:

$$\frac{\partial f_n}{\partial t} + \frac{\vec{p}}{ma^2}\cdot\frac{\partial f_n}{\partial \vec{x}} - m\sum_{\tilde{n}\neq n}\frac{\partial \phi_{\tilde{n}}}{\partial \vec{x}}\cdot\frac{f_n}{\partial \vec{p}} = 0$$

integrate out UV modes

- apply window function that cuts off $k > \Lambda$ and expand $f(\vec{x}, \vec{p})$ in moments of $\vec{p} \Rightarrow$ fluid equations for δ and v^i

- equations for higher moments suppressed by mean free path

- DM moves slowly compared to $H \Rightarrow$ effective mean free path $v/H \sim 1/k_{NL}$, so fluid description valid

smoothed overdensity

smoothed fluid velocity

$$\dot{\delta} + \frac{1}{a}\partial_i((1+\delta)\, v^i\,) = 0$$

$$\dot{v}^i + H v^i + \frac{1}{a} v^j \partial_j v^i + \frac{1}{a}\,\partial_i \phi \; = -\frac{1}{a\bar{\rho}}\,\partial_j [\tau^{ij}]_\Lambda$$

$\partial_i \partial^i \phi \sim \Omega_m \delta$

effective stress tensor

induced stress tensor

- smoothed effective stress tensor $[\tau^{ij}]_\Lambda$ is a function of long modes
- expansion in perturbations and k/k_{NL} (constrained by symmetry)

bulk viscosity

shear viscosity

$$\tau^{ij} = \bar{p}\delta^{ij} + \bar{\rho}\left(c_1 \delta + \frac{c_2}{aH} \partial_k v^k \right)\delta^{ij} + \bar{\rho}\,\frac{c_3}{aH}\,\partial^{(i} v^{j)} + \Delta\tau^{ij} + \ldots$$

speed of sound

stochastic stress

- parameters encode expectation values of short modes in the presence of long modes

perturbation theory

- linear equations relate δ to $\partial_i v^i$, so viscosity and sound speed terms are degenerate at one loop

- fluid equations at one loop

$$\dot{\delta} + \frac{1}{a}\partial_i((1+\delta)v^i) = 0$$

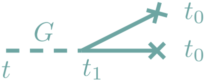
$$\partial_i \dot{v}^i + H\partial_i v^i + \frac{1}{a}\partial_i(v^j\partial_j v^i) + \frac{1}{a}\partial^2\phi = -\frac{1}{a}c_s^2\partial^2\delta + \frac{1}{a\bar{\rho}}\partial_i\partial_j\Delta\tau^{ij}$$

perturbation theory

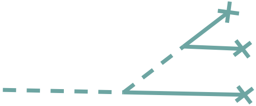
- expand in perturbations: $\delta = \delta^{(1)} + \delta^{(2)} + \delta^{(3)} + \delta^{(\mathrm{ct})}$
- higher order terms sourced with Green's function
- diagrammatic expansion

$\delta^{(1)} =$

$\delta^{(2)} =$



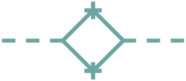
$\delta^{(3)} =$



loops + counterterms

- $\langle\delta\delta\rangle = \langle\delta^{(1)}\delta^{(1)}\rangle + \langle\delta^{(2)}\delta^{(2)}\rangle + \langle\delta^{(1)}\delta^{(3)}\rangle + 2\langle\delta^{(1)}\delta^{(\mathrm{ct})}\rangle$
- from equations of motion: counterterm proportional to linear field, $\delta^{(\mathrm{ct})} \sim \left(\frac{k}{k_{NL}}\right)^2 \delta^{(1)}$

$\langle\delta^{(1)}\delta^{(1)}\rangle$



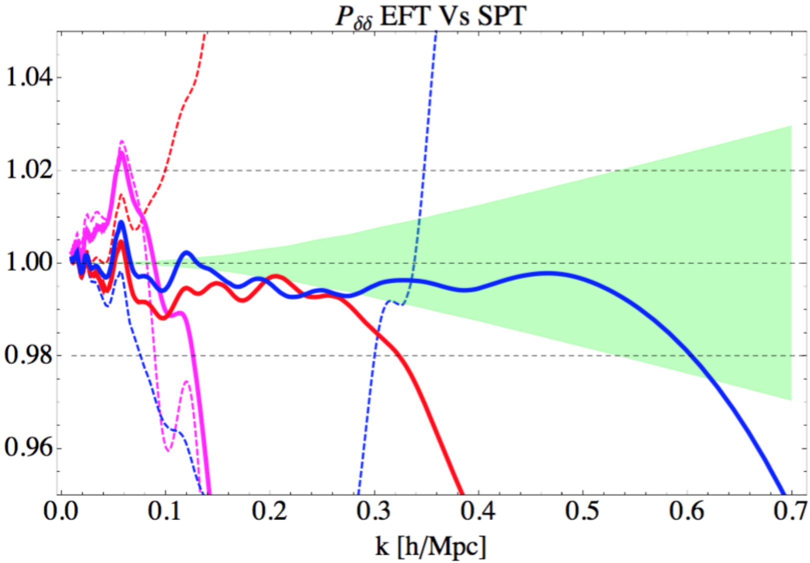
$\langle\delta^{(2)}\delta^{(2)}\rangle$

$\langle\delta^{(3)}\delta^{(1)}\rangle$

$\langle\delta^{(1)}\delta^{(\mathrm{ct})}\rangle$

- Smoothed fields δ and v^i depend on smoothing scale Λ
- Λ-dependence in loops canceled by Λ-dependence of counter-term c_s

dark matter results at two loops



Carrasco, Foreman, Green, Senatore (2013)

status of EFT of LSS

- redshift-space distortion
  Senatore, Zaldarriaga (2014)
- bias
  Senatore (2014), Angulo, Fasiello, Senatore, Vlah (2015)
- higher redshifts
  Foreman, Senatore (2014)
- higher correlation functions
  Angulo, Foreman, Schmittful, Senatore (2014)
- baryons
  Lewandowski, AP, Senatore (2014)

the problem with baryons: astrophysical processes

- various baryon processes modify the matter power spectrum by $> 1\%$ on relevant scales

- baryon effects include: star formation, SN feedback, AGN feedback

van Daalen, Schaye, Booth, and Dalla Vecchia (2011)

a fluid description for baryons?

- complicated to simulate baryon physics, analytical treatment possible?

- baryons explode and stream out: effective fluid?

- very non-relativistic, even when hot and mass density lost at smoothing scale negligible

- in a cluster baryons and dark matter occupy the same regions

a simple modification of EFT

- generalize to 2 particle species interacting only via gravity with relative densities $w_b = \Omega_{\mathrm{baryon}}/\Omega_{\mathrm{m}}$, $w_c = \Omega_{CDM}/\Omega_{\mathrm{m}}$

$$\dot{\delta}_{\mathbf{c}} = -\frac{1}{a}\partial_i((1+\delta_{\mathbf{c}}){v_{\mathbf{c}}}^i)$$

$$\dot{\delta}_{\mathbf{b}} = -\frac{1}{a}\partial_i((1+\delta_{\mathbf{b}}){v_{\mathbf{b}}}^i)$$

$$\partial_i \dot{v}^i_{\mathbf{b}} + H\partial_i v^i_{\mathbf{b}} + \frac{1}{a}\partial_i(v^j_{\mathbf{b}}\partial_j v^i_{\mathbf{b}}) + \frac{1}{a}\partial^2\phi = -\frac{1}{a}\partial_i\left(\partial\tau_\rho\right)_{\mathbf{b}}{}^i + \frac{1}{a}\partial_i(\gamma)_{\mathbf{b}}{}^i$$

$$\partial_i \dot{v}^i_{\mathbf{c}} + H\partial_i v^i_{\mathbf{c}} + \frac{1}{a}\partial_i(v^j_{\mathbf{c}}\partial_j v^i_{\mathbf{c}}) + \tfrac{1}{a}\partial^2\phi = -\frac{1}{a}\partial_i\left(\partial\tau_\rho\right)_{\mathbf{c}}{}^i + \frac{1}{a}\partial_i(\gamma)_{\mathbf{c}}{}^i$$

$\partial^2\phi \sim \omega_b\delta_b + \omega_c\delta_c$

effective stress tensor

momentum exchange part, only affects stochastic term

counter-terms

- at one loop, four possible parameters:

response to velocity gradients and star-formation physics

gravitationally induced speed of sound

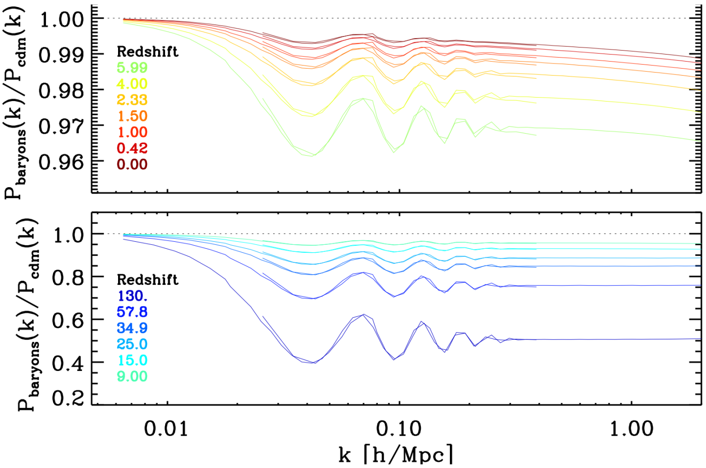
$$\partial_i \left(\partial\tau_\rho\right)^i_{\mathbf{b}} - \partial_i(\gamma)^i_{\mathbf{b}} \sim c^2_{\mathbf{b},g} \left(w_{\mathbf{c}}\partial^2\delta_{\mathbf{c}} + w_{\mathbf{b}}\partial^2\delta_{\mathbf{b}}\right) + c^2_{\mathbf{b},v}\, \partial^2\delta_{\mathbf{b}}$$

$$\partial_i \left(\partial\tau_\rho\right)^i_{\mathbf{c}} - \partial_i(\gamma)^i_{\mathbf{c}} \sim c^2_{\mathbf{c},g} \left(w_{\mathbf{c}}\partial^2\delta_{\mathbf{c}} + w_{\mathbf{b}}\partial^2\delta_{\mathbf{b}}\right) + c^2_{\mathbf{c},v}\, \partial^2\delta_{\mathbf{c}}$$

perturbation theory

- basis of adiabatic (total matter) $\delta_A = w_{\mathbf{c}}\delta_{\mathbf{c}} + w_{\mathbf{b}}\delta_{\mathbf{b}}$ and isocurvature modes: $\delta_I = \delta_{\mathbf{c}} - \delta_{\mathbf{b}}$
- from linear equations, $\delta_I^{(1)} \sim$ const and $\delta_A^{(1)}(k, a) \sim D(a)$, linear growth factor for total matter
- at $z = 0$, $\delta_I/\delta_A \sim 10^{-2} \rightarrow$ isocurvature mode suppressed, can neglect in loops
- because isocurvature loops neglected, counterterms needed for only adiabatic diagrams, so only two c_s parameters come in

isocurvature mode

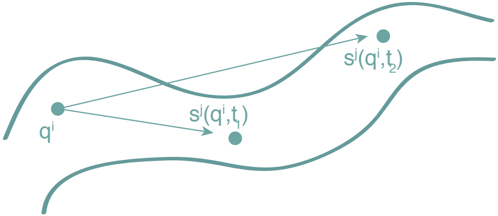


Angulo, Hahn, Abel (2013)

resummation of bulk flows

- one fluid: large bulk flow does not affect equal time correlators because of equivalence principle
- two fluids: argument still holds for adiabatic mode
- but - relative motion between baryons and DM gives dynamical effect in all observables
- it is an IR effect, so we can resum it

bulk flows



- perturbation theory done in Eulerian space: fixed reference frame
- Lagrangian approach: track fluid flow using displacement $\vec{s}$ from inital position $\vec{q}$
  Matsubara (2008)
- displacements affect matter density: $\delta(\vec{k},t) = \int d^3q \ \exp[-i\vec{k}\cdot(\vec{q}+\vec{s})]$
- effects of large displacements break perturbation theory in Eulerian theory, but are perturbative in Lagrangian theory

IR resummation: hybrid approach

- IR-resummed Eulerian correlator $\xi(\vec{r})$ at a given order in perturbations is sum of $\xi(\vec{q})$ at lower orders weighted by probability to be displaced from $\vec{q}$ to $\vec{r}$

- from Lagrangian approach, $P(\vec{r}|\vec{q}) \sim \int d^3k \; e^{-i\vec{k}\cdot(\vec{q}-\vec{r})} e^{-(\vec{k}\cdot\Delta\vec{s}_1)^2}$

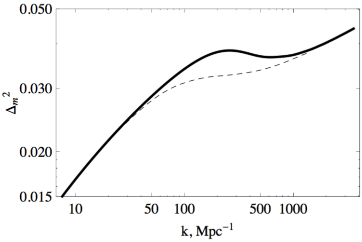
- resums leading effect of long displacements on density, remaining effect is perturbative

$$\xi|_{\epsilon_\delta^N}(\vec{r}, t_1, t_2) \;= \sum_{j=0}^{N} \int d^3q \;\; P|_{\leq \epsilon_\delta^{N-j}}(\vec{r}|\vec{q}, t_1, t_2) \;\; \xi|_{\epsilon_\delta^j, \epsilon_s^j}(\vec{q})$$

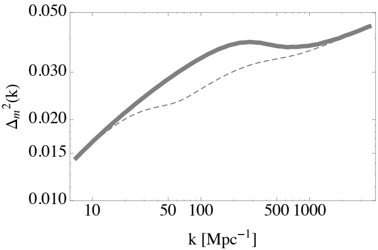
- corresponds to perturbative corrections to Zeldovich approximation

effect of relative velocity on BAO peak

- modify IR resummation to include baryons: large effect in cross-correlation
- relative velocity effect large at $z \sim 40$ and leads to a breaking of perturbation theory
- EFT provides a consistent perturbative scheme, with higher order corrections



Tseliakhovich and Hirata (2010)



Lewandowski, AP, Senatore (2014)

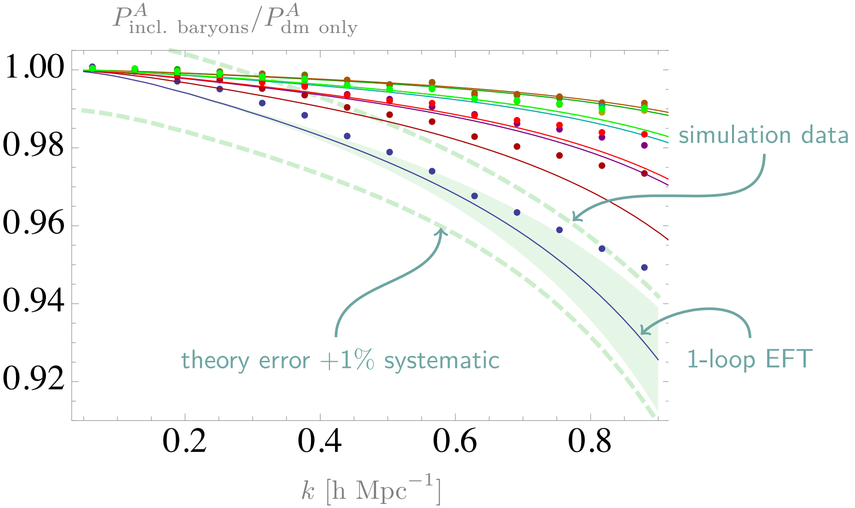
results

comparison to simulations

$$P^{\mathbf{c}}(k) = P^{\mathbf{c}}{}_{11}(k) + P^A_{1-\text{loop}}(k) - 2(2\pi)\left(\bar{c}^2_A(a_0) + w_{\mathbf{b}}\bar{c}^2_I(a_0)\right)k^2 P^A_{11}(k)$$
$$P^{\mathbf{b}}(k) = P^{\mathbf{b}}{}_{11}(k) + P^A_{1-\text{loop}}(k) - 2(2\pi)\left(\bar{c}^2_A(a_0) - w_{\mathbf{c}}\bar{c}^2_I(a_0)\right)k^2 P^A_{11}(k)$$

- $\bar{c}^2_A = c^2_{\text{no baryon}} + w_b \Delta\bar{c}^2_A$
- $\Delta\bar{c}^2_A$: effect of baryons on total matter speed of sound, determine by matching to $P^A/P^A_{\text{dm only}}$
- $\bar{c}^2_I$: effect of having 2 species, determine by matching to P^b/P^A

EFT results at one loop: determining $\Delta\bar{c}_A^2$

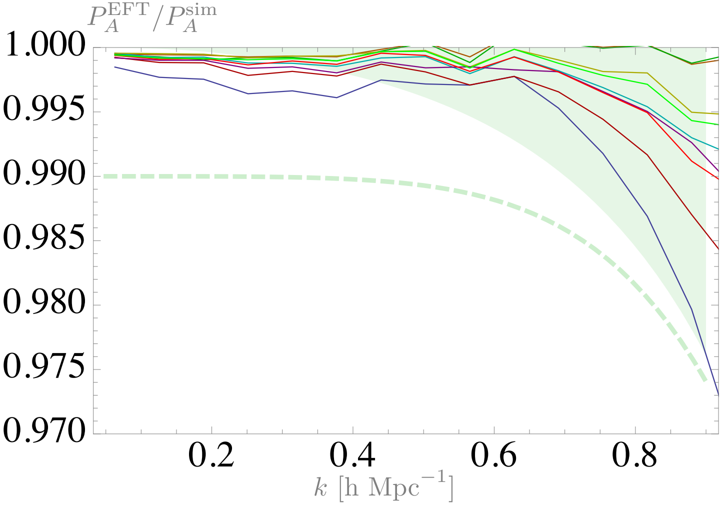


$\Delta\bar{c}_A^2 = -.3$ to $-3\ k_{NL}^{-2}$, $k_{NL} = 4.0\ \text{Mpc}^{-1}$

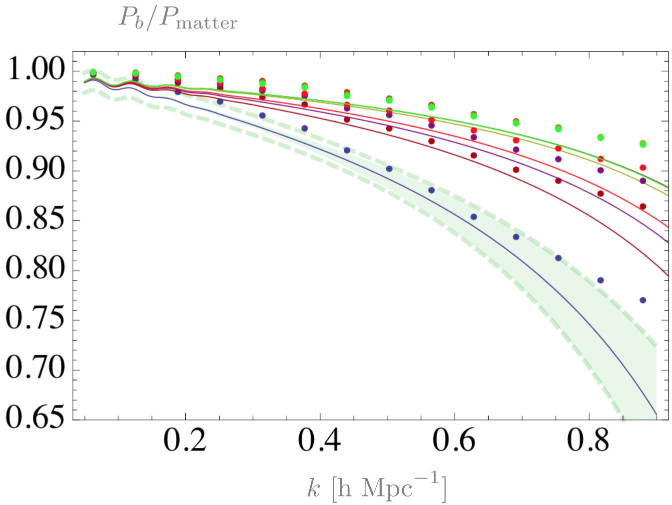
simulations used

Simulation	$w_b \Delta \bar{c}_A^2 \left[k_{\rm NL}^{-2}\right]$	$\bar{c}_I^2 \left[k_{\rm NL}^{-2}\right]$	Description
AGN	0.42 ± 0.13	-2.17 ± 0.24	Includes AGN (in addition to SN feedback)
DBLIMFV1618	0.24 ± 0.08	-1.25 ± 0.24	Top-heavy IMF at high pressure, extra SN energy wind velocity
NOSN	0.063 ± 0.017	N/A	No SN energy feedback
NOSN_NOZCOOL	0.059 ± 0.033	-0.72 ± 0.2	No SN energy feedback and cooling assumes primordial abundances
NOZCOOL	0.10 ± 0.034	N/A	Cooling assumes primordial abundances
WDENS	0.16 ± 0.025	-1.06 ± 0.24	Wind mass loading and velocity depend on gas density (same SN energy as REF)
WML1V848	0.15 ± 0.025	-0.96 ± 0.24	Wind mass loading $\eta = 1$, velocity $v_w = 848$ km/s (same SN energy as REF)
WML4	0.093 ± 0.034	-0.72 ± 0.24	Wind mass loading $\eta = 4$ (twice the SN energy as REF)
REF	0.093 ± 0.034	-0.77 ± 0.29	Reference simulation

EFT results at one loop

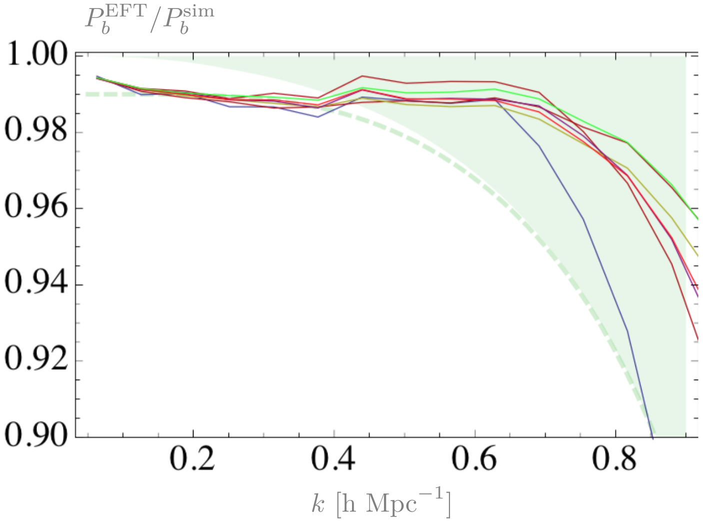


EFT results at one loop: determining $\bar{c}_I^2$



$\bar{c}_I^2 = -0.7$ to $-2\ k_{\rm NL}^{-2}$, $k_{NL} = 4.0\ {\rm Mpc}^{-1}$

EFT results at one loop: P_b



effect of baryons

- c_s^2 parameters order one: baryon effects well approximated by EFT
- effect of baryons captured in just one extra parameter, and a simple functional form $k^2 P_{11}^A(k)$
- very different baryon effects (difficult to simulate) correspond to different sound speeds

concluding remarks

- LSS can potentially beat CMB constraints on primordial parameters
- we must improve constraints by increasing $k_{\max}$
- including baryons an important part of matching theory to observations